EV-A (Advanced Course in Experimental Physics):
Lasers, Atomic Physics and Quantum Optics
Universität Erlangen–Nürnberg
Winter Term 2018/2019

Exercise Sheet 6 (29.11.2018)

Lecture: Tuesday, Thursday 12:00 - 14:00, lecture hall HH
Tutorial: Wednesday: 10:00 - 12:00, SRLP 0.179 | Thursday 08:00 - 10:00, SR 00.732, SR 01.779, SRLP 0.179
Hompage: www.qoqi.nat.fau.de/teaching/lectures

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1) Polarization, susceptibility and refractive index in steady state

From the lecture we know (see §3.I.) that the macroscopic polarization P is related to the driving electromagnetic field E by the relation $Pe^{i\omega t} = \epsilon_0 \chi E_0 e^{i\omega t}$, with χ the dielectric susceptibility.
(a) Show that $P = \frac{N_{at}}{V} d_{eg}(u + iv)$, with u and v the x- and y-components of the Bloch-vector, respectively. To that aim demonstrate that $Re(Pe^{i\omega t}) = \frac{N_{at}}{V} \langle\psi|d|\psi\rangle$, with $Re(A)$ the real-part of A and $\langle\psi|d|\psi\rangle$ the expectation value of d for atoms in the state ψ, as calculated in the lecture.
(b) Calculate the susceptibility and the index of refraction in steady state. To that purpose derive the stationary solutions u_{st}, v_{st} and w_{st} for u, v and w from the Bloch equations with damping.

2) Typical values of a helium-neon laser

In this exercise, we want to study typical quantities of a helium-neon (HeNe) laser in operation.
(a) In general, how does the mean photon number in the resonator $\bar{n}_{ph}$ and the atomic inversion n_{st} depend on the pump rate R? Draw a plot of $\bar{n}_{ph}(R)$ and $n_{st}(R)$.

A HeNe laser oscillates at $\lambda = 0.63\mu m$, the transversal and longitudinal damping rates are given by $\gamma' = 10^9/s$ and $\gamma = 5 \cdot 10^7/s$, respectively, the damping rate of a typical HeNe-resonator is $\gamma_c = 10^7/s$, while the "Rabi frequency per photon" $g = d_{eg}/\hbar\sqrt{\hbar\omega/\epsilon_0 V}$ corresponds to $g \approx 10^5/s$.
(b) With which rate R do we need to pump in order to enter the lasing regime (assuming resonant excitation)?
(c) Calculate the atomic inversion if the laser is above the lasing threshold.
(d) For a pump rate of $R_1 = 10^{16}/s$ ($R_2 = 10^{12}/s$), calculate the mean photon number $\bar{n}_{ph}$ in the resonator and the outcoupled laser power P, assuming that losses and transmission are equally contributing to γ_c.

3) Relaxation oscillations

In the lecture, the following rate equations for the atomic inversion in a laser medium $n(t)$ and the number of photons in the resonator mode $n_{ph}(t)$ were derived from the Maxwell-Bloch equations:

$$\begin{aligned} \dot{n}_{ph}(t) &= -(\gamma_c - \kappa n(t))n_{ph}(t) \\ \dot{n}(t) &= -\kappa n_{ph}(t)n(t) - \gamma n(t) + R\,. \end{aligned} \qquad (1)$$

Here, γ_c is the damping rate of the resonator, γ is the natural line width of the upper laser level $|e\rangle$ (= longitudinal damping rate of the Bloch vector), κ is the transition rate for stimulated emission (= Einstein B coefficient) and

R is the pump rate.

(a) Show that in steady state and above the lasing threshold, the atomic inversion is given by $n_{st} = \gamma_c/\kappa$ and the number of photons in the resonator is given by $\bar{n}_{ph} = (R - R_{th})/\gamma_c$, with $R_{th} = \gamma_c\gamma/\kappa$.

(b) Show that above the lasing threshold in close vicinity to steady state the ansatz $n_{ph}(t) = \bar{n}_{ph} + \delta n_{ph}(t)$, $n(t) = n_{st} + \delta n(t)$ and $\rho = R/R_{th}$ lead to the following coupled system of differential equations for $\delta n_{ph}(t)$ and $\delta n(t)$

$$\begin{aligned} \delta\dot{n}_{ph}(t) &= (\rho - 1)\gamma\delta n(t) \\ \delta\dot{n}(t) &= -(\gamma + \kappa\bar{n}_{ph})\delta n(t) - \gamma_c\delta n_{ph}(t)\,. \end{aligned}$$

(c) Show that this leads to the following uncoupled second order differential equations for $\delta n_{ph}(t)$ and $\delta n(t)$

$$\begin{aligned} \delta\ddot{n}_{ph}(t) &= -\rho\gamma\delta\dot{n}_{ph}(t) - (\rho - 1)\gamma\gamma_c\delta n_{ph}(t) \\ \delta\ddot{n}(t) &= -\rho\gamma\delta\dot{n}(t) - (\rho - 1)\gamma\gamma_c\delta n(t)\,. \end{aligned} \tag{2}$$

(d) Solve (2) with the ansatz $x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$ (with $x(t) = \delta n_{ph}(t)$ and $x(t) = \delta n(t)$ respectively). Under which circumstances above the lasing threshold ($\rho > 1$) can you find relaxation oscillations?

Consider a laser cavity with $\gamma_c = 10^7\,s^{-1}$, a pump power of $P = 5W$ (with $P = R \cdot const.$), a pump threshold $P_{th} = 0.5W$ and the following gain media: Nd:YLF ($\tau = 480\mu s$), Nd:YVO_4 ($\tau = 50\mu s$) and Ti:sapphire ($\tau = 3.2\mu s$).

(e) Verify for all three cases whether relaxation oscillations will occur.

(f) What pump powers would be necessary to suppress the relaxation oscillations?