EV-A (Advanced Course in Experimental Physics):
Lasers, Atomic Physics and Quantum Optics
Universität Erlangen–Nürnberg
Winter Term 2018/2019

Exercise Sheet 3 (8.11.2018)

Lecture: Tuesday, Thursday 12:00 - 14:00, lecture hall HH
Tutorial: Wednesday: 10:00 - 12:00, SRLP 0.179 | Thursday 08:00 - 10:00, SR 00.732, SR 01.779, SRLP 0.179
Hompage: www.qoqi.nat.fau.de/teaching/lectures

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1) Transverse and longitudinal modes of different types of resonators

Consider a symmetric resonator with two identical mirrors $R_1 = R_2 = R$ at positions z_1 and z_2, respectively, separated by a distance L.
(a) Sketch the beam radius of the TEM_{00} fundamental mode at the location of the mirror (i.e., the *spot size* on the mirrors) for a fixed wavelength λ as a function of $g = 1 - L/R$ (i.e. for different types of resonators) within the range $0 \leq g^2 \leq 1$. For which resonator type is the *spot size* maximal and for which is it minimal?
(b) Sketch the beam waist parameter ω_0 of the TEM_{00} fundamental mode for a fixed wavelength λ as a function of $g = 1 - L/R$. For which type of resonator is the beam waist maximal and for which minimal?
(c) According to the answers to a **(a)** and **(b)**, what are the advantages and disadvantages for (i) confocal resonators, (ii) nearly planar resonators and (iii) almost concentric resonators when used as laser resonators? In your answer take into account the spectral properties of the resonators and if they can be easily aligned or not.
(d) Consider a general Hermite-Gaussian mode of the resonator (with arbitrary transverse mode numbers n, m). Calculate the phase accumulated by the beam along the z-axis (including the Gouy phase $\psi(z)$) when travelling from z_1 to z_2.
(e) Which phase condition has to be fulfilled by a beam after one round trip to be sustained by the resonator? Using this requirement, show that the resonance frequencies of the resonator are given by

$$\nu_{qnm} = \frac{c}{2L}\left[q + (n+m+1)\frac{\arccos(\pm\sqrt{g_1 g_2})}{\pi}\right],$$

where q denotes the longitudinal mode number.
(f) For the three resonator types listed in **(c)**: Are the transverse modes degenerate? Calculate the free spectral range of the TEM_{00} mode.

2) Natural linewidth of a two level atom

Einstein's rate equations for the occupation numbers N_1 and N_2 of a two level atom with lower level E_1 and upper level E_2, respectively, are given by (see lecture):

$$\begin{aligned}\dot{N}_2 &= -AN_2 - BN_2u(\omega) + BN_1u(\omega)\\ \dot{N}_1 &= -BN_1u(\omega) + AN_2 + BN_2u(\omega)\end{aligned}$$

where A and B are the Einstein coefficient for spontaneous emission and stimulated emission/stimulated absorption, respectively, and $u(\omega)$ is the energy density of the electromagnetic field at the frequency $\omega = (E_2 - E_1)/\hbar$.

(a) Solve the rate equations for N_2 in the case of $u(\omega) = 0$. What does $u(\omega) = 0$ mean? How can one interpret the result? How is the natural lifetime τ of the level E_2 related to the Einstein coefficient A?

The electrical field strength, which is emitted by the atom, is given by: $\vec{E}(t) = \vec{E}_0 e^{-t/(2\tau)} e^{-i\omega t}$.
(b) Show that the decrease of the field strength leads to a line broadening in the frequency spectrum of the emitted radiation and calculate its precise shape.
(c) Calculate the natural linewidth (in nm) of the following transitions of a helium-neon laser: (i) $3s_2 \to 2p_4$: $\lambda = 632.8\,\text{nm}$, (ii) $2s_2 \to 2p_4$: $\lambda = 1152.3\,\text{nm}$, (iii) $3s_2 \to 3p_4$: $\lambda = 3391.3\,\text{nm}$. The lifetimes τ of the levels involved are: $2s_2$: 96 ns, $3s_2$: 100 ns, $2p_4$: 22 ns, $3p_4$: 9.8ns.

3) Damping in optical resonators - the Fabry-Perot-Etalon

A Fabry-Perot-Etalon (FPE) consists of two identical parallel mirrors forming an optical resonator, with a medium between the mirrors of refraction index n. Such a device is used to filter an incident light field, as only modes of specific frequencies are transmitted by the FPE. The spectral width $\Delta\omega$ of the transmission spectrum of the FPE is governed by the losses of the resonator. Those losses are determined by the transmission and absorption coefficient T and A, respectively, indicating the intensity transmitted and absorbed by one of the mirrors upon reflection. After switching off the external field at $t = t_0$, the transmitted intensity decays exponentially:

$$I_{tr}(t) = I_{tr}(t_0) e^{-\frac{t-t_0}{\tau_c}} . \tag{1}$$

(a) How is the decay time τ_c linked to the spectral width $\Delta\omega$ of the transmitted light field?
(b) The transmission spectrum of an FPE is given by

$$S(\omega) = \frac{T^2}{(1-R)^2} \frac{1}{\left(1 + \frac{4R}{(1-R)^2} \sin^2\left(\frac{\omega n L \cos\theta}{c}\right)\right)}, \tag{2}$$

with θ denoting the angle of the light to the optical axis **within** the FPE (see Fig. 1 below). Show that, neglecting absorbtion, $S(\omega)$ can be rewritten as

$$S(\omega) = \frac{1}{1 + \left(\frac{2}{\pi} F \sin\left(\frac{\omega L \sqrt{n^2 - \sin^2(\alpha)}}{c}\right)\right)^2}, \tag{3}$$

with the Finesse F and α denoting the incident angle of the light field **outside** the FPE (see Fig. 1). Sketch the resulting spectrum as a function of ω.
(c) How can $S(\omega)$ be simplified in the proximity of a single transmission peak? Using this approximation derive an explicit expression for the spectral width $\Delta\omega$.
(d) Calculate the decay time τ_c for an FPE with a reflectance of the two mirrors of $R = 0.969$. The length is $L = 0.5\text{mm}$ and the refractive index of the medium between the mirrors is $n = 1.5$. The incident light field can be considered to impinge parallel to the optical axis.
(e) In a realistic scenario, the incoming light field cannot be treated as an optical ray, i.e., a plane wave, resulting in a decreased effective Finesse F_{eff} due to a non-zero divergence Θ of the incoming beam:

$$\frac{1}{F_{eff}} = \sqrt{\frac{1}{F^2} + \frac{1}{F_{div}^2}} \tag{4}$$

with the divergence finesse $F_{div} = \frac{\lambda}{nL\tan^2(\Theta)}$. Estimate values for Θ resulting in a significant broadening of the spectral width due to the divergence of the incoming beam. Assume a typical optical wavelength of $\lambda = 600\text{nm}$.

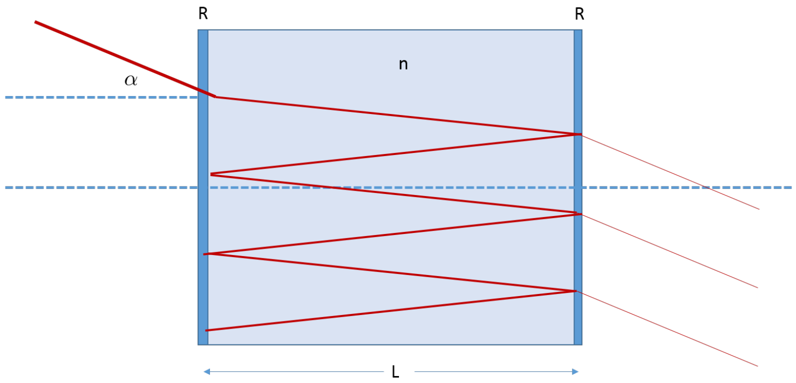

Abbildung 1: Fabry-Perot-Etalon