§ 3 Propagation of waves in optical media

I. Wave equation

Maxwell's equations in vacuum (1873):

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

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I. Wave equation

***Maxwell's equations* in matter (1873):**

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{D}/\epsilon_0 = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu\mu_0 \left(\vec{j} + \frac{\partial \vec{D}}{\partial t} \right)$$

Additionally there are the constitutive relations :

$$\vec{D} = \epsilon_0 \vec{E}_D + \vec{P} = (1+\chi)\epsilon_0 \vec{E} = \epsilon\epsilon_0 \vec{E} \quad \text{with} \quad \vec{P} = \epsilon_0 \chi \vec{E}_\mathrm{D} = \frac{N}{V}\alpha \vec{E}_\mathrm{D}$$

$\vec{B} = \mu\mu_0 \vec{H}$ (usually $\mu \approx 1$)

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I. Wave equation

wave equation in vacuum in absence of charges and currents:

$$\Delta \vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0$$

with $\frac{1}{c^2} = \epsilon_0 \mu_0$

For a monochromatic field with harmonic time dependence $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})e^{-i\omega t}$

Helmholtz equation :

$$\Delta \vec{} + k^2 \vec{E} = 0$$

with $k^2 = \frac{\omega^2}{c^2}$

Wave equation in matter in absence of charges and currents:

$$\Delta \vec{D} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{D} = 0$$

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II. Elementary Solutions

Solution of Helmholtz equation in cartesian coordinates:

$$\vec{E}(\vec{r},t) = \vec{E}_0 \vec{\epsilon} e^{i(\vec{k}\vec{r}-\omega t)}$$

$$\vec{k} \cdot \vec{E} = 0$$

$$\vec{k} \times \vec{B} = \frac{\omega}{c^2}\vec{E}$$

$\Longrightarrow$ $\vec{k} \perp \vec{E} \perp \vec{B} = 0$

(transversal wave)

Solution of Helmholtz equation in spherical coordinates:

for small solid angle:

$$E(\vec{r},t) = E_0 \frac{e^{i(\vec{k}\vec{r}-\omega t)}}{|\vec{k}\vec{r}|}$$